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Characterizing stress fields in the upper crust using joint orientation distributions

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Abstract

Linear elastic fracture mechanics predicts that joint orientation is controlled by the stress field in which the joints propagate. Thus joint sets are effective proxies for stress trajectories during joint growth. Complexity in joint orientation indicates stress trajectory variability, a phenomenon quantified using an eigenvalue method that measures dispersion of joint normal vectors (i.e. poles) around the mean vector. Ratios between the eigenvalues of a joint orientation tensor give the clustering strength (ζ) and the shape factors (γ) of the distribution.

A joint set that forms in a relatively isotropic rock subject to a rectilinear stress field should exhibit strong clustering and small random orientation variation that can be described by the Fisher statistical model. However, most joint orientation distributions in bedded rocks have non-random variation, greater in strike than in dip. This relative stability of the vertical stress orientation is strongest when joints are bounded by bedding interfaces, reflecting the tendency for deflection in the local stress field arising from the growth of side cracks, joint segments and adjacent joints in joint zones. Even when joint growth across bedding interfaces indicates negligible strength anisotropy, joint orientation distributions reflect less joint–joint interaction during vertical growth than during horizontal growth. As strike variation grows due to the presence of a non-rectilinear stress field, the orientation data better fit a Kent statistical model. Joint sets formed during fold development or in rocks with irregular bedding boundaries are more weakly clustered with Fisher-like orientation distributions. Orientation distributions for joint sets formed throughout a stress rotation have Kent-like shapes that indicate the magnitude of stress trajectory variation and clustering strength that depends on the joint density at each increment of the stress rotation.

Keywords: Joint set; Rectilinear stress field; Joint orientation distribution; Overburden stress; Tectonic stress; Fisher statistical model; Kent statistical model

1. Introduction

Joints provide records of stress orientation at the time of propagation (Pollard and Segall, 1987). Provided the stress difference during joint propagation was sufficient to allow two relatively closely spaced joints to pass each other without deflection (i.e. Olson and Pollard, 1989), joint orientation data indicate the extent to which the principal stress trajectories remained parallel across a sample volume (Engelder and Geiser, 1980). If a rectilinear stress field governed propagation throughout the affected rock volume, poles to individual joints cluster strongly about the mean pole of the joint set. Alternatively, joint poles cluster weakly around the mean pole of the joint set if the principal stress trajectories were non-parallel spatially and/or changed orientation over time.

Joint patterns in the foreland portions of some mountain belts have well-defined orientation modes suggesting that portions of the upper crust are subject to rectilinear stress fields (e.g. Melton, 1929; Babcock, 1973; Hancock et al., 1984; Dunne and North, 1990) (Fig. 1A). Furthermore, the contemporary tectonic stress fields in the upper crust of eastern North America and northwestern Europe appear rectilinear to a first approximation (Zoback, 1992). However, joint patterns in orogenic forelands more often have weak or multiple orientation modes suggesting that rectilinear stress fields in the upper crust are the exception rather than the rule (Fig. 1B and C). In these latter cases, confusion arises when defining a joint set based on strike alone, because weak orientation modes can arise from a non-rectilinear stress field (e.g. Parker, 1942; Verbeek and

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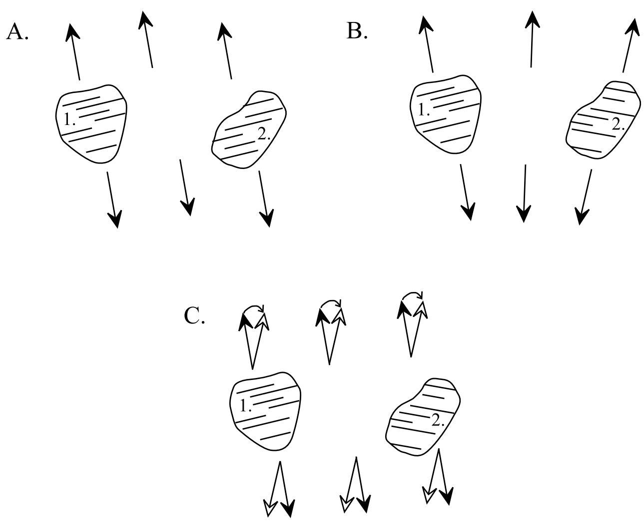

Fig. 1. Diagrams of joint patterns at outcrops separated by covered intervals. (A) A hypothetical regional joint set has the same orientation from outcrops 1 to 2. (B) A hypothetical regional joint set changes in orientation from outcrops 1 to 2 due to a radial stress field. (C) Two hypothetical joint sets develop in the region due to temporal variation in the stress field.

Grout, 1983; Laubach and Lorenz, 1992; Arlegui and Simon, 2001) (Fig. 1B) or through time during the regional rotation of a rectilinear stress field (e.g. Engelder, 2004) (Fig. 1C).

An analysis of a regional stress field using dikes around Spanish Peaks shows that stress trajectories were not parallel at the scale of the dike set (Odé, 1957). Rock discontinuities like bedding, faults, joints and inclusions like concretions also perturb regional stress fields (e.g. Olson and Pollard, 1989; Rawnsley et al., 1992; McConaughy and Engelder, 1999). In this context, the question that we raise and address is whether stress fields in the upper crust are *ever* rectilinear even at the scale of a sample volume the size of an outcrop. If not, we must conclude that as in the rock adjacent to the Spanish Peaks stocks (Odé, 1957), localized stress heterogeneities always add complexity to the stress field, and hence fracture pattern. Taken at face value, orientation data from the World Stress Map suggest that stress trajectories at scales larger than the outcrop are not strictly parallel even in places like the upper crust of eastern North America (Zoback, 1992).

Our approach to answering the question about the scale of rectilinear stress fields in the upper crust is to use the dispersion of orientation for a joint set as a proxy for the degree of variability of stress trajectories in a rock volume as a function of some combination of space and time. All joints in a set formed in a rectilinear stress field and hosted by an isotropic material should be parallel, and, if measured perfectly, their normal vectors (i.e. the projections of the poles in the lower hemisphere) should plot at a single point on a spherical projection. For any real joint set, however, the poles to joints plot in a region on the sphere, with their point concentration decreasing away from a mean vector (Fig. 2).

As long as joints propagate without being deflected by the presence of neighbors, we presume that joint sets formed in a rock displaying material isotropy and subject to a rectilinear stress field will have the strongest clustering about the mean orientation. In this case, joint set data would be subject only to random variation in orientation arising from the propagation of side cracks (e.g. Lacazette and Engelder, 1992) causing the overlap of joint segments (e.g. Hodgson, 1961) and the growth of joint zones (e.g. Engelder, 1987). These factors plus measurement error should yield a data set with dip dispersions equal to strike dispersions. Poles to such joints sets would fit the Fisher statistical model, where dispersion about the mean vector is assumed to be circular, i.e. unvarying with direction (Fisher, 1953).

When three-dimensional fracture variation is quantified in the literature, the Fisher model is most often applied under the assumption that joint dispersion is primarily governed by random variation (e.g. Priest, 1993; Song et al., 2001; Kemeny and Post, 2003; Engelder and Delteil, 2004). However, inspection of joint distributions on stereographic projections reveals that joint dispersion is not always random and that, specifically, strike dispersion generally exceeds dip dispersion (Fig. 2; Table 1). When joint dispersion is not random, the tightness, shape and orientation of the vector concentration reflect stress field complexity at the time of jointing and overlap of joint segments, as well as the compass's precision and the skill of the operator. By comparing joint distributions from various tectonic settings and noting the differences between strike and dip dispersions, we can assess the extent to which horizontal tectonic and vertical gravitational stresses remain rectilinear during jointing.

We apply the eigenvalue ratio method of Woodcock (1977) to quantify the distribution of joint poles in a set (Fig. 3). To assess the effect of increasing stress complexity on joint set dispersion, we analyze joint orientation data from horizontal sedimentary rocks, horizontal sedimentary rocks subject to a stress field rotation, folded rocks, and folded rocks subject to stress field rotation (Table 1). To demonstrate the effect of rock properties on joint set dispersion, we compare joint orientation data from black shale whose minor anisotropy arises from a pervasive compaction under overburden stress with data from bedded sediments whose major anisotropy arises from a change in lithology. In the former case, vertical joint propagation encounters little change in fracture strength whereas in the latter case there is a large contrast in fracture strength from bed to bed. From these results, we conclude that eigenvalue ratios of joint normal vectors are convenient for assessing stress complexity during fracturing, determining whether a joint sample represents one (unimodal) or more (multimodal) sets, and deciding which probability density model for the orientation data (e.g. Fisher, 1953 or Kent, 1982) applies to the joint set.

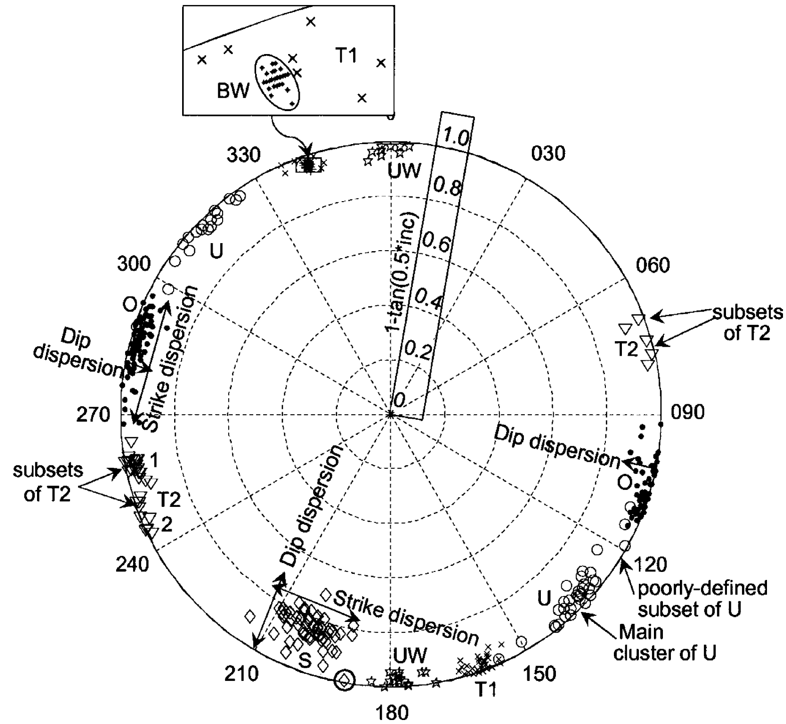

Fig. 2. Equal-angle, lower-hemisphere stereographic projection of normal vectors to joints from selected sets in Table 1 showing a sample of the distribution shapes taken on by joint data. Fisher (1953) distribution is valid for joint sets whose strike dispersions are not statistically different from their dip dispersions, as is the case for the S data set. Commonly, joint set strike dispersion exceeds dip dispersion, resulting in an elliptical distribution better characterized by the Kent (1982) model, for example the O joint set. For near-vertical joint sets such as O, the two halves of the dip dispersion plot 180° apart in strike. Symbols: asterisk, BW; star, UW; times, T1; circle, U; dot, O; triangle, T2; diamond, S; location abbreviations are the same as Table 1; Inc, inclination of the joint pole. On the equal-angle plot, the shape of the distribution remains the same from the perimeter of the circle to the center, but its size per arc length on the projection decreases towards the center. The datum circled on the southern perimeter of the projection is the one removed in the Split Mountain sensitivity test documented in Table 1. Note that subset 1 of T2 contains more joints than subset 2.

2. Quantifying dispersion about a mean joint pole

The orientation of a joint plane can be represented by a unit vector parallel to its pole. This unit vector has three orthogonal components, the direction cosines, which are projected onto the north, east and down axes. The vector sum magnitude, R, of n unit vectors (i.e. joint poles) with direction cosines $\begin{bmatrix} u_i & v_i & w_i \end{bmatrix}$ is:

$$R = \sqrt{\left(\sum_{i=1}^{n} u_i\right)^2 + \left(\sum_{i=1}^{n} v_i\right)^2 + \left(\sum_{i=1}^{n} w_i\right)^2}. \qquad (1)$$

The mean pole of the joint set is a vector given by the direction cosines (e.g. Fisher, 1953):

$$\begin{bmatrix} u_{\text{mean}} & v_{\text{mean}} & w_{\text{mean}} \end{bmatrix} = \begin{bmatrix} \dfrac{\sum_{i=1}^{n} u_i}{R} & \dfrac{\sum_{i=1}^{n} v_i}{R} & \dfrac{\sum_{i=1}^{n} w_i}{R} \end{bmatrix}. \qquad (2)$$

Watson (1966) determined that the moment of inertia was an analog to the clustering of unit vectors. The moment of inertia of a set of masses depends on the size of the masses (m) and their distances from the rotational axis (r):

$$I = \sum_{i=1}^{n} m_i r_i^2. \qquad (3)$$

By representing each unit vector intersection with the sphere as a unit mass, the problem reduces to each datum's distance from three perpendicular rotational axes, which the direction cosines of the unit vectors determine. In this way, Watson (1966) constructed the normalized orientation tensor (T) to quantify the clustering of directional data:

$$T = \frac{1}{n} \begin{bmatrix} \left(\sum_{i=1}^{n} u_i^2\right) & \left(\sum_{i=1}^{n} u_i v_i\right) & \left(\sum_{i=1}^{n} u_i w_i\right) \\ \left(\sum_{i=1}^{n} v_i u_i\right) & \left(\sum_{i=1}^{n} v_i^2\right) & \left(\sum_{i=1}^{n} v_i w_i\right) \\ \left(\sum_{i=1}^{n} w_i u_i\right) & \left(\sum_{i=1}^{n} w_i v_i\right) & \left(\sum_{i=1}^{n} w_i^2\right) \end{bmatrix}. \qquad (4)$$

The eigenvectors of T (Table 1) define a coordinate system where ξ_1 is the axis in the direction that minimizes the moment of inertia and ξ_3 is in the direction that maximizes the moment of inertia and ξ_2 is perpendicular to ξ_1–ξ_3 plane (Watson, 1966). The ξ_1 axis is in the direction of the mean vector or mean pole to the joint set. The eigenvalues (λ_1, λ_2, λ_3) of T represent relative

Table 1
Joint data sources and summary of clustering analysis

Location	*N*	Rock	Env	log (λ_1/λ_2)	log (λ_2/λ_3)	γ	ζ	ξ_1		ξ_2		ξ_3		Formation	Age
								Dec	Inc	Dec	Inc	Dec	Inc		
Bizanos Wall (BW) (Engelder and Delteil, 2004)	50	U	H	9.41	1.82	5.17	11.2	341	04	133	85	251	02		
Taughannock Falls, NY 070 strike (T1)	47	U	H	5.66	0.790	7.16	6.45	160	02	251	25	067	65	Geneseo	D
Corning, NY (CO)	42	U	H	5.69	0.657	8.67	6.35	059	01	328	10	154	80	Rhinestreet	D
Watkins Glen, NY (W)	59	B	H	5.27	1.00	5.26	6.28	246	03	156	00	061	87	Ithaca	D
Rock Camp, WV (R)	22	B	HR	3.91	2.23	1.75	6.15	138	02	229	06	33	84	Bluefield	M
Taughannock Falls, NY 345 strike (T2)	40	U	HR	4.76	1.14	4.16	5.90	256	02	346	17	158	73	Geneseo	D
Lilstock, U.K. (L) (Engelder and Peacock, 2001)	47	B	F	5.23	0.604	8.65	5.83	212	5	113	58	305	31	Lias	J
Union, WV (UW)	29	U	FR	5.42	0.402	13.5	5.82	178	01	268	11	85	79	Bluefield	M
Chocin, Czech Republic (C) (Engelder and Delteil, 2004)	101	B	H	3.85	1.96	1.97	5.81	346	01	256	01	124	88	Jizera	K
Elk Basin, WY (E) (Engelder et al., 1997)	37	B	F	5.40	0.370	14.6	5.77	276	08	013	41	177	48	Eagle	K
Fort Smith, AR (F)	37	B	H	4.99	0.772	6.47	5.76	071	02	161	18	335	72	Atoka	P
Octavia, OK (O)	180	B	F	4.74	0.827	5.73	5.57	286	01	016	04	177	86	Jackfork	P
Huntingdon, PA (H) (Ruf et al., 1998)	78	B	FR	4.52	0.833	5.43	5.35	143	10	051	07	287	78	Brallier	D
Usti, Czech Republic (U) (Engelder and Delteil, 2004)	56	U	HR	3.88	1.26	3.08	5.14	134	02	043	05	247	85	Jizera	K
Split Mountain, UT (S) (Silliphant et al., 2002)	69	B	F	4.76	0.229	20.8	4.99	201	18	312	49	097	36	Chinle	T
Split Mountain, UT less one joint (Silliphant et al., 2002)	68	B	F	4.96	0.0471	105	5.00	201	18	294	11	054	69	Chinle	T

Locations and external data sources are noted. Location abbreviations used in the text are given in parentheses. Abbreviations: *N*, sample size; Rock Behavior (Rock): B, bed-bounded joints; U, unbounded joints. Geological Environment (Env): H, horizontal strata; F, folded strata; R, stress rotation observed in outcrop by presence of fringe cracks, sheared joints, or by joints that curve along strike; λ_1, λ_2, λ_3 are eigenvalues of T; γ, shape factor; ζ, strength factor; ξ_1, ξ_2, ξ_3 are eigenvectors; dec, declination (lower hemisphere); inc, inclination (positive down). When ξ_2 plunges shallowly, the primary dispersion is in the strike direction. The orientation distribution takes on an increasingly circular form as ξ_2 increases in plunge from 0 to 45°, and again becomes more elliptical as it increases in plunge from 45 to 90°. When ξ_2 plunges steeply, the primary dispersion is in the dip direction. Ages: D, Devonian; M, Mississippian; P, Pennsylvanian; T, Triassic; J, Jurassic; K, Cretaceous.

concentrations or clustering in the respective directions ξ_1, ξ_2 and ξ_3.

Applying this approach to determining the clustering around the mean pole of a joint set requires one modification. For near-vertical features like joints, the normal vectors for two features with a small intersection angle but opposite dip directions project to opposite axes, resulting in a non-representative vector sum near zero. To circumvent this problem, the orientation data should be rotated arbitrarily so that they are all in the same dip quadrant. After quantifying the clustering, the mean vector and cluster description should be rotated to the original coordinates to establish true clustering characteristics.

The ratios between λ_1, λ_2 and λ_3 indicate the shape and concentration of the normal vector distribution, and are displayed concisely in a Woodcock (1977) eigenvalue ratio plot (Fig. 3). For example, if λ_1 is large compared with λ_2 and λ_3 and the value of R approaches n, the sample has a single mode (Watson, 1966). Thus, as λ_1/λ_2 increases, the sample tends to a single mode, whereas when λ_2/λ_3 increases, the cloud of normal vectors stretches from a circle to an ellipse to a girdle (Woodcock, 1977). The term γ quantifies the shape of the distribution much like a Flinn (1962) diagram, and is equivalent to the slope of the line connecting the origin of the plot to a data point. The tightness of the individual data in the distribution about the mean vector or girdle is quantified by the strength (ζ) factor (Woodcock, 1977):

$$\zeta = \log\left(\frac{\lambda_1}{\lambda_3}\right). \tag{5}$$

For a given shape factor, joint orientation distributions with large ζ have a smaller magnitude of strike and dip variation than those with small ζ.

3. Joint sets

New and published joint orientation data from sedimentary rocks in varied geological settings in the United States and Europe were analyzed by the eigenvalue method (Table 1). The outcrops are identified according to whether vertical joint growth was bed-bounded, whether the beds were folded, and whether outcrop evidence suggests stress rotation during joint set formation. These joint sets are found in a variety of tectonic positions including regional cross-fold sets (CO, W, T2, F, O), fold-parallel sets (L, E, H), sets transected by fold axes (UW, R, S, T1), and indeterminate sets (C, U). The T1, T2, CO, U, and UW joints are not confined to individual beds, but at bedding interfaces the CO and UW joints often change dip. The T1 set predates the T2 set at that sample site (Engelder et al., 2001).

To summarize the 14 joint data sets described here, ζ ranges from 5 to 6.5 and γ ranges from 1.75 to more than 20 (Table 1). For most of the joint samples, ξ_1 is equivalent to the mean vector of joint normals, ξ_2 plunges shallowly and ξ_3 plunges steeply. No joint set data yield an infinite strength (ζ), which arises when a precisely measured joint set propagates in an isotropic rock subject to a rectilinear stress field (Fig. 3). Further, cluster strength in the natural joint sets has been observed to be sensitive to n, in that, as the sample size increases, the given shape becomes better defined (Fig. 4). This indicates that small joint data samples may not randomly sample the underlying joint population to characterize its basic attributes. For orientation data, a minimum sample size of 30 has been cited as necessary to produce statistically significant results (Fisher et al., 1987).

4. Statistical criteria for joint set dispersion

For more rigorous statistical modeling of joint set orientations in three dimensions, the probability density distributions developed for orientation data from unimodal populations can be applied (Fisher, 1953; Kent, 1982). While the Fisher (1953) model, with two parameters, is computationally simpler than the five-parameter Kent (1982) model, it requires rotational symmetry about the mean. Geological and engineering studies have often modeled joint sets using the Fisher distribution (e.g. Priest, 1993; Song et al., 2001; Kemeny and Post, 2003; Engelder and Delteil, 2004); however, any joint set with statistically greater variation in either the strike or dip direction does not

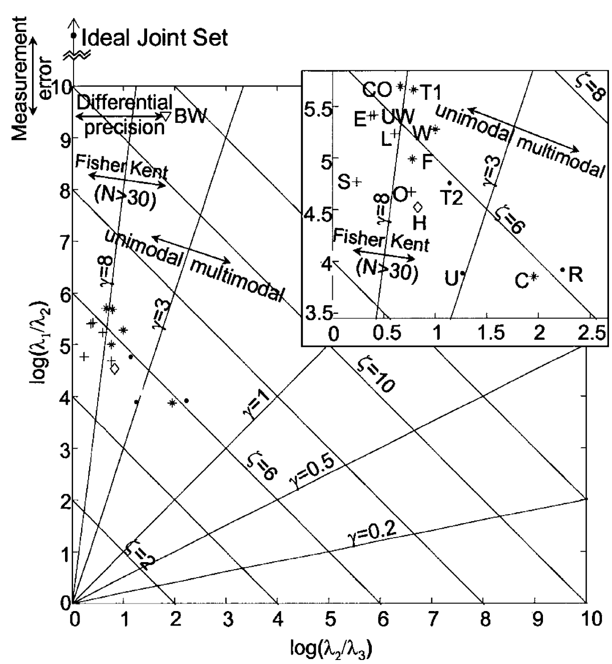

Fig. 3. Woodcock (1977) logarithmic eigenvalue ratio plot of 15 data sets summarized in Table 1. Inset of plot shows location labeling (see Table 1). N = sample size. Symbols: triangle, Bizanos Wall (BW); asterisk, horizontal rock; dot, horizontal rock with rotated stress field; plus, folded rock; diamond, folded rock with rotated stress field; γ, shape factor; ζ, strength factor.

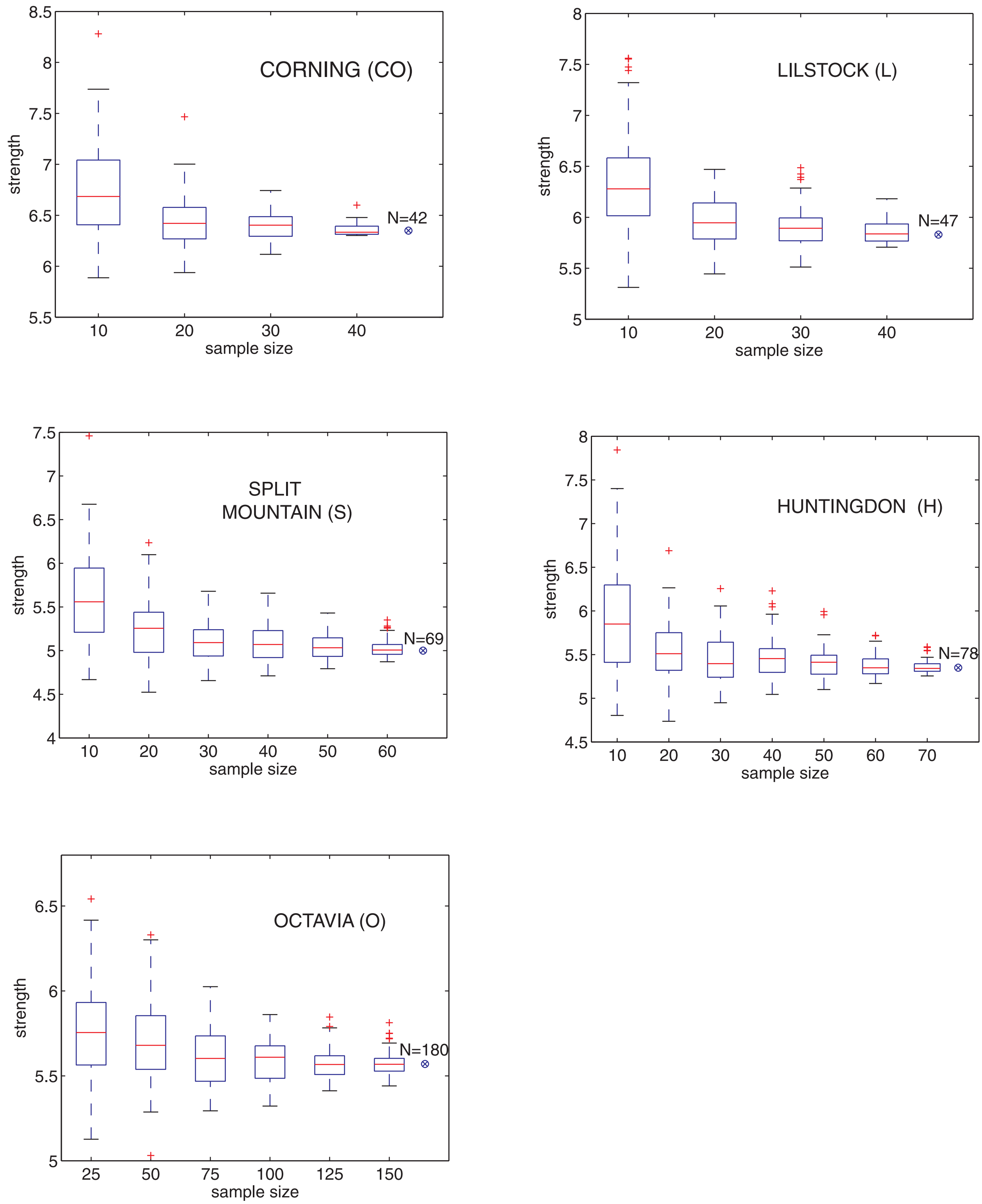

Fig. 4. Box plot of strength factors for 100 random samples of increasing size from the total samples for locations CO, L, S, H, and O (Table 1), selected to represent a variety of distribution strengths and sample sizes. The circle on the right edge of the plot represents the whole sample distribution strength.

meet this criterion (Fig. 2). The Kent (1982) model, with an end member case being the Fisher distribution, can accommodate elliptical dispersion of joint orientations about the mean (Peel et al., 2001).

The shape factor is an effective tool for screening which parametric distribution is appropriate for each joint set. As rotational symmetry implies that $\lambda_1 > \lambda_2 = \lambda_3$, a pure Fisher (1953) distribution plots on the y-axis of the eigenvalue ratio diagram. A Fisher distribution indicates the presence of a rectilinear stress field on the scale of a sample volume that is usually an outcrop. Based on the goodness of fit test statistic (Fisher et al., 1987; Kent, personal communication), the null hypothesis that the data fit the Fisher distribution cannot be rejected for joint sets with shape factors above 8 for sample sizes of 30 or more (e.g. CO, L, UW, E and S in Fig. 3). For $\gamma < 8$, there is statistical significance to the elongation of the distribution in the direction of ξ_2. As n increases, it becomes increasingly difficult to fit the Fisherian constraint at larger values of γ. This n required to disprove rotational symmetry is directly proportional to γ because, for example, dispersion with an aspect ratio of 1.1 can be proven non-circular with fewer data than can dispersion with an aspect ratio of 1.01. For small ζ, λ_2, λ_3, ξ_2, and ξ_3 are especially sensitive to outliers (e.g. S, Table 1). For the values of γ between 3 and 8, the Kent model applies to most joint sets with sample sizes from 30 to more than 100 (e.g. T1, W, F, O, H, BW). The elliptical dispersion within joint sets in this range indicates the absence of a rectilinear stress field on the scale of the outcrop. In the interval of shape between 3 and 4.5, some joint distributions technically fit a unimodal statistical model, but appear as two distinct sets separated by a small angle in the outcrop (e.g. T2) that should be separated before statistical analysis. For values of γ below 3, the sets diverge from a unimodal distribution (e.g. C, U, R). Either two joint sets with different mean vectors begin to emerge, indicating a distinct change in the stress field between jointing events (e.g. Younes and Engelder, 1999), or the joint set clusters around a small circle, which suggests that fracturing occurred continuously during a stress field rotation or that the stress field curved across the sampling domain.

The selection of the incorrect probability distribution model for the data can introduce significant error into a simulation of fracture orientation (Fig. 5). For example, applying the Fisher (1953) model to five joint sets that have elliptical dispersion underestimates the vector dispersion in the ξ_2 (declination) direction and overestimates the dispersion in the ξ_3 (inclination) direction as determined with the Kent (1982) model. The errors are symmetrical across the ξ_1–ξ_2 and ξ_1–ξ_3 planes, so the percent error is doubled over the whole distribution.

5. Discussion

While the orientation distribution for a rectilinear joint set should plot at infinity on the y-axis of a Woodcock plot, no data collected under field conditions ever would. This point is highlighted by the data from BW, a single plane measured 50 times with a Freiberger geological compass (Fig. 3; Table 1). Although the plane remains the same, the compass and operator do not enable identical measurements, so the data have finite strength. If the compass-induced error was equal in declination and inclination, the BW data would plot on the y-axis of the diagram. However, the precision of the compass is greater in declination than in inclination, giving the BW distribution a finite shape. So, depending on the instrument and operator, data collected from a joint set formed in an isotropic rock under a rectilinear stress field may have at least a slightly elliptical distribution.

The Geneseo Formation, Devonian black shale of the Appalachian Plateau, New York, has negligible mechanical anisotropy relative to joint growth as indicated by the vertical propagation of joints through slight compaction anisotropy (Engelder et al., 2001; Lash et al., 2004). T1 joints in the Geneseo Formation propagated in pre- or early Alleghanian time with a standard deviation of 2.4° for outcrop average strike at 14 locations in a 1400 km^2 area (Lash et al., 2004). This is equivalent to a circular variance of 0.00474 on a scale of 0–1. Such widespread consistency in joint attitude suggests the presence of a rectilinear stress field across the region during the propagation of the T1 set. Indeed, at Taughannock Falls, the T1 set has the largest cluster strength ($\zeta = 6.45$) of any outcrop-scale joint sets measured (Table 1). This suggests that stress trajectories during T1 fracturing approached maximum parallelism possible for outcrop-scale volume of rock because the rock is so transparent to direction of joint growth. Yet, despite the tight cluster, and little evidence for deformation-associated dispersion of the T1 joints regionally or locally, their dispersion at the scale of the Taughannock Falls outcrop is greater in strike than in dip (Table 1), indicating that T1 propagation was not entirely controlled by random processes of joint growth in a rectilinear stress field. In particular, subtle stress heterogeneity is reflected in horizontal trajectories that are less consistent than vertical trajectories.

The subtle stress heterogeneity in the direction does not automatically signal a non-rectilinear remote stress field. One source of heterogeneity arises from the propagation of side cracks (e.g. Lacazette and Engelder, 1992) causing the overlap of joint segments (e.g. Hodgson, 1961) and the growth of joint zones (e.g. Engelder, 1987). The T1 outcrop extends for several hundred meters where the trace of 'individual' joint traces revealed overlapping crack segments. These fluid-driven joints (e.g. Lacazette and Engelder, 1992) are more likely to propagate toward each other in the horizontal direction, leading to a greater possibility of joint deflection *à la* Olson and Pollard (1989) in the horizontal direction. T1 sits so close to a Kent distribution ($\gamma = 7.16$) that there is every reason to nominate these data for an outcrop propagating in a regional rectilinear stress field.

At other outcrops in the Appalachian Basin (CO, W) where sediments are interbedded siltstones and shale, cross-fold joint clustering is strong ($\zeta > 6$), characteristic of formation in a rectilinear stress field. Where the joints cross bedding surfaces in the Rhinestreet Formation (CO), they maintain a distribution nearly identical to that for jointing in the more homogeneous black shale of the Geneseo Formation (T1). Where joints are bed-bounded in the Ithaca Formation (W), however, γ is smaller than for either T1 or CO (Table 1). The effect of the mechanical bed boundaries is to limit vertical growth and thereby prevent joint–joint interaction that might mask a 2D rectilinear stress field on vertical planes. In contrast, horizontal growth allows for joint–joint interaction in that direction, thus accenting the non-rectilinear nature of the horizontal stress field and this is reflected in greater dispersion in joint strikes than in joint dips (Table 1). We note that W and T2 are cross-fold joint sets subject to stress rotation with time (i.e. Younes and Engelder, 1999).

Orientation distributions for joint sets formed throughout a stress rotation have shape values that indicate the amount of rotation, and strength values that depend on how uniform the joint density is over the distribution. In rocks that were subject to a varying stress field orientation during fracturing, the largest joint set ζ is found at Rock Camp (R) where a temporal stress field rotation is indicated by the overprinting of joints with slickenlines and clockwise fringe cracks. The density of joint orientations in the R set remains constant throughout the distribution, resulting in a large ζ. Here small γ indicates an arcuate distribution of joint strikes due to temporal variation in a rectilinear stress field during fracturing rather than pervasive instantaneous fracturing throughout a non-rectilinear stress field (e.g. Spraggins and Dunne, 2002). At Union (U), an outcrop proximal to R, the joint orientation distribution has a large γ (Fig. 2), which indicates that jointing occurred at a single instance in the stress rotation recorded at R. A discrete stress rotation of small magnitude (larger γ) during Alleghanian shortening is observable in T2 cross-fold joints (Fig. 2). The relative complexity of the horizontal stress trajectories between the foreland Arkoma Basin and the Ouachita fold and thrust belt is captured in the γ values for Fort Smith (F) and Octavia (O) joint sets, which demonstrate that the former experienced a smaller stress rotation during jointing (Table 1). In the T2 joints, one joint subset is more frequent than the other and there is a break in the orientation distribution between the sets (Fig. 2), so the strength of that distribution is smaller than the strength at R (Table 1). At Usti (U) (Fig. 2) and at Chocin (C), one joint subset is poorly represented (Fig. 2), causing ζ at these locations to be considerably smaller than for R and T2 joints (Table 1).

As a group, folded joint sets and joint sets formed during fold development have larger values of γ than joints found in any other structural setting, with most fitting the Fisherian statistical constraint. The orientation distributions of two joint sets that formed concurrently with folding (L, E) have ζ with values near 5.8, while another fold-related set (H) has a smaller value (Table 1; Fig. 3). The variation in joint dip that occurs within these sets is an indication of the layer's temporal rotation with respect to the vertical principal stress during the fracturing interval. H does not have the circular normal vector dispersion of the other fold-related sets but shows fringe crack development, indicating horizontal stress rotation in addition to a relative vertical stress rotation. The joints at Split Mountain (S) formed in the fluvial Chinle Formation, and comprised a circular orientation distribution that most likely is influenced by random processes. The joints with shallow dips that increase the dip dispersion in the S sample were mechanically influenced during fracturing by the position of non-horizontal channel boundaries.

In summary, the eigenvalue ratios for joint sets, and therefore the cluster strength (ζ) and shape (γ), can be used to infer whether the remote stress field in the upper crust was

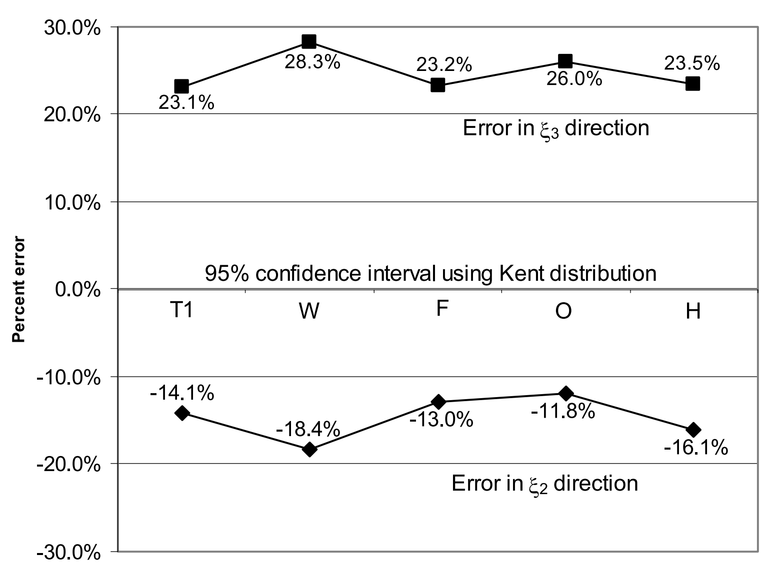

Fig. 5. Error introduced into the probability density distribution by incorrectly applying the Fisher (1953) model instead of the Kent (1982) model, the joint sets T1, W, F, O, and H (Table 1). The joint distributions selected for the analysis have the five largest shape factors for distributions that do not fit the Fisher (1953) model, and therefore have the highest likelihood of being mistakenly forced into the Fisher (1953) orientation distribution.

rectilinear during joint propagation. Joint sets with the largest distribution strengths and relatively large shape factors indicate the presence of a rectilinear stress field at least on the scale of outcrops. A small γ for a joint orientation distribution within an outcrop generally indicates the rotation of the horizontal stress during fracturing. Joint sets that develop in non-rectilinear stress fields present during folding have the greatest distribution shape factors, but smaller strength factors than joint sets formed in horizontal rocks. Irregular bed boundaries can contribute to joint dip dispersion, also leading to a large γ, small ζ distribution.

Joint orientation distributions in horizontal beds, especially where bedding did not arrest vertical growth, indicate that the vertical dimension of the fracture is more consistently oriented than the strike dimension. This result is interpreted to mean that the local stress due to growth of side cracks, joint segment overlap and joint zones causes scatter in joint orientation in horizontal planes. Bed rotation during folding or irregular mechanical boundaries causes a deviation from the rectilinear stress field common to outcrops in undeformed forelands.

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